# The msm Package

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Title Multi-state Markov and hidden Markov models in continuous time

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**Description** Functions for fitting general continuous-time Markov and hidden Markov multi-state models to longitudinal data. Both Markov transition rates and the hidden Markov output process can be modelled in terms of covariates. A variety of observation schemes are supported, including processes observed at arbitrary times, completely-observed processes, and censored states.

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# **R** topics documented:

1atrixExp	2
neur	3
oot.msm	4
OS	6
oef.msm	7
rudeinits.msm	8
eltamethod	9
matrix.msm	11
ev	12
azard.msm	13
eart	14
mm-dists	14
ogLik.msm	17
nedists	18
1sm	20
dds.msm	29
exp	30

plot.msm
pmatrix.msm
pmatrix.piecewise.msm
prevalence.msm
psor
qmatrix.msm
qratio.msm
sim.msm
simmulti.msm
sojourn.msm
statetable.msm
msm.summary
surface.msm
tnorm
totlos.msm
transient.msm
viterbi.msm

# Index

53

MatrixExp Matrix exponential

# Description

Calculates the exponential of a square matrix.

# Usage

MatrixExp(mat, t = 1, n = 20, k = 3, method="pade")

# Arguments

mat	A square matrix
t	An optional scaling factor for the eigenvalues of mat
n	Number of terms in the series approximation to the exponential
k	Underflow correction factor, for the series approximation
method	"pade" for the Pade approximation, or "series" for the power series approximation.

aneur

#### Details

The exponential E of a square matrix M is calculated as

$$E = U \exp(D) U^{-1}$$

where D is a diagonal matrix with the eigenvalues of M on the diagonal,  $\exp(D)$  is a diagonal matrix with the exponentiated eigenvalues of M on the diagonal, and U is a matrix whose columns are the eigenvectors of M.

This method of calculation is used if M has distinct eigenvalues. I If M has repeated eigenvalues, then its eigenvector matrix may be non-invertible. In this case, the matrix exponential is calculated using the Pade approximation defined by Moler and van Loan (2003), or the less robust power series approximation,

$$\exp(M) = I + M + M^2/2 + M^3/3! + M^4/4! + \dots$$

For a continuous-time homogeneous Markov process with transition intensity matrix Q, the probability of occupying state s at time u + t conditional on occupying state r at time u is given by the (r, s) entry of the matrix  $\exp(tQ)$ .

The implementation of the Pade approximation was taken from JAGS by Martyn Plummer (http: //www-fis.iarc.fr/~martyn/software/jags).

The series approximation method was adapted from the corresponding function in Jim Lindsey's R package rmutil (http://popgen.unimaas.nl/~jlindsey/rcode.html).

## Value

The exponentiated matrix  $\exp(mat)$ .

#### References

Cox, D. R. and Miller, H. D. The theory of stochastic processes, Chapman and Hall, London (1965) Moler, C and van Loan, C (2003). Nineteen dubious ways to compute the exponential of a matrix, twenty-five years later. SIAM Review 45, 3-49. At http://epubs.siam.org/sam-bin/dbg/article/41801

aneur

Aortic aneurysm progression data

## Description

This dataset contains longitudinal measurements of grades of aortic aneurysms, measured by ultrasound examination of the diameter of the aorta.

#### Usage

data (aneur)

#### Format

A data frame containing 4337 rows, with each row corresponding to an ultrasound scan from one of 838 men over 65 years of age.

ptnum(numeric)Patient identification numberage(numeric)Recipient age at examination (years)diam(numeric)Aortic diameterstate(numeric)State of aneurysm.

The states represent successive degrees of aneurysm severity, as indicated by the aortic diameter.

State 1	Aneurysm-free	< 30 cm
State 2	Mild aneurysm	30-44 cm
State 3	Moderate aneurysm	45-54 cm
State 4	Severe aneurysm	> 55 cm

683 of these men were aneurysm-free at age 65 and were re-screened every two years. The remaining men were aneurysmal at entry and had successive screens with frequency depending on the state of the aneurysm. Severe aneurysms are repaired by surgery.

#### Source

The Chichester, U.K. randomised controlled trial of screening for abdominal aortic aneurysms by ultrasonography.

#### References

Jackson, C.H., Sharples, L.D., Thompson, S.G. and Duffy, S.W. and Couto, E. Multi-state Markov models for disease progression with classification error. *The Statistician*, 52(2): 193–209 (2003)

Couto, E. and Duffy, S. W. and Ashton, H. A. and Walker, N. M. and Myles, J. P. and Scott, R. A. P. and Thompson, S. G. (2002) *Probabilities of progression of aortic aneurysms: estimates and implications for screening policy* Journal of Medical Screening 9(1):40–42

boot.msm

*Bootstrap resampling for multi-state models* 

#### Description

Draw a number of bootstrap resamples, refit a msm model to the resamples, and calculate statistics on the refitted models.

#### Usage

```
boot.msm(x, stat=pmatrix.msm, B=500, file=NULL)
```

#### boot.msm

#### Arguments

Х	A fitted msm model, as output by msm.
stat	A function to call on each refitted msm model. By default this is pmatrix.msm, returning the transition probability matrix in one time unit. If NULL then no function is computed.
В	Number of bootstrap resamples.
file	Name of a file in which to save partial results after each replicate. This is saved using save and can be restored using load, producing an object called boot.list containing the partial results.

#### Details

The bootstrap datasets are computed by resampling independent transitions between pairs of states (for non-hidden models without censoring), or independent patient series (for hidden models or models with censoring).

Confidence intervals or standard errors for the corresponding statistic can be calculated by summarising the returned list of B replicated outputs. This is currently implemented for the transition probability matrix, see pmatrix.msm. At the moment, for other outputs, users will have to write their own code to summarise the output of boot.msm.

Most of **msm**'s output functions present confidence intervals based on asymptotic standard errors calculated from the Hessian. These are expected to be underestimates of the true standard errors (Cramer-Rao lower bound). Bootstrapping may give a more accurate estimate of the uncertainty.

All objects used in the original call to msm which produced x, such as the qmatrix, should be in the working environment, or else boot.msm will produce an "object not found" error. This enables boot.msm to refit the original model to the replicate datasets.

If stat is NULL, then B different msm model objects will be stored in memory. This is unadvisable, as msm objects tend to be large, as they contain the original data used for the msm fit, so this will be wasteful of memory.

To specify more than one statistic, write a function consisting of a list of different function calls, for example,

stat = function(x) list (pmatrix.msm(x, t=1), pmatrix.msm(x, t=2))

#### Value

A list with B components, containing the result of calling function stat on each of the refitted models. If stat is NULL, then each component just contains the refitted model. If one of the B model fits was unsuccessful and resulted in an error, then the corresponding list component will contain the error message.

## Author(s)

C.H.Jackson <chris.jackson@imperial.ac.uk>

## References

Efron, B. and Tibshirani, R.J. (1993) An Introduction to the Bootstrap, Chapman and Hall.

bos

#### See Also

pmatrix.msm, totlos.msm

#### Examples

```
## Not run:
    ## Psoriatic arthritis example
    data(psor)
    psor.q <- rbind(c(0,0.1,0,0),c(0,0,0.1,0),c(0,0,0.1),c(0,0,0,0))
    psor.msm <- msm(state ~ months, subject=ptnum, data=psor, qmatrix = psor.q, covariates = ~
    ## Bootstrap the baseline transition intensity matrix. This will take a long time.
    q.list <- boot.msm(psor.msm, function(x)x$Qmatrices$baseline)
    ## Manipulate the resulting list of matrices to calculate bootstrap standard errors.
    apply(array(unlist(q.list), dim=c(4,4,5)), c(1,2), sd)
    ## Similarly calculate a bootstrap 95% confidence interval
    apply(array(unlist(q.list), dim=c(4,4,5)), c(1,2), function(x)quantile(x, c(0.025, 0.975))
    ## Bootstrap standard errors are larger than the asymptotic standard errors calculated from
    psor.msm$QmatricesSE$baseline
## End(Not run)
```

bos

Bronchiolitis obliterans syndrome after lung transplants

#### Description

A dataset containing histories of bronchiolitis obliterans syndrome (BOS) from lung transplant recipients. BOS is a chronic decline in lung function, often observed after lung transplantation. The condition is classified into four stages of severity: none, mild, moderate and severe.

#### Usage

data(bos)

#### Format

A data frame containing 638 rows, grouped by patient, including histories of 204 patients. The first observation for each patient is defined to be stage 1, no BOS, at six months after transplant. Subsequent observations denote the entry times into stages 2, 3, 4, representing mild, moderate and severe BOS respectively, and stage 5, representing death.

ptnum	(numeric)	Patient identification number
time	(numeric)	Months after transplant
state	(numeric)	BOS state entered at this time

#### Details

The entry time of each patient into each stage of BOS was estimated by clinicians, based on their history of lung function measurements and acute rejection and infection episodes. BOS is only

6

## coef.msm

assumed to occur beyond six months after transplant. In the first six months the function of each patient's new lung stabilises. Subsequently BOS is diagnosed by comparing the lung function against the "baseline" value.

## Source

Papworth Hospital, U.K.

## References

Heng. D. et al. (1998). Bronchiolitis Obliterans Syndrome: Incidence, Natural History, Prognosis, and Risk Factors. Journal of Heart and Lung Transplantation 17(12)1255–1263.

coef.msm Extract model coefficients

## Description

Extract the estimated log transition intensities and the corresponding linear effects of each covariate.

## Usage

```
## S3 method for class 'msm':
coef(object, ...)
```

#### Arguments

object	A fitted multi-state model object, as returned by msm.
• • •	(unused) further arguments passed to or from other methods.

#### Value

If there is no misclassification, coef.msm returns a list of matrices. The first component, labelled logbaseline, is a matrix containing the estimated transition intensities on the log scale with any covariates fixed at their means in the data. Each remaining component is a matrix giving the linear effects of the labelled covariate on the matrix of log intensities.

For misclassification models, coef.msm returns a list of lists. The first component, Qmatrices, is a list of matrices as described in the previous paragraph. The additional component Ematrices is a list of similar format containing the logit-misclassification probabilities and any estimated covariate effects.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

#### See Also

msm

crudeinits.msm

#### Description

Calculates crude initial values for transition intensities by assuming that the data represent the exact transition times of the Markov process.

#### Usage

```
crudeinits.msm(formula, subject, qmatrix, data=NULL, censor=NULL, censor.states=NUI
```

#### Arguments

formula	A formula giving the vectors containing the observed states and the correspond- ing observation times. For example,
	state ~ time
	Observed states should be in the set 1, $\ldots$ , n, where n is the number of states.
subject	Vector of subject identification numbers for the data specified by formula. If missing, then all observations are assumed to be on the same subject. These must be sorted so that all observations on the same subject are adjacent.
qmatrix	Matrix of indicators for the allowed transitions. An initial value will be esti- mated for each value of qmatrix that is greater than zero. Transitions are taken as disallowed for each entry of qmatrix that is 0.
data	An optional data frame in which the variables represented by subject and state can be found.
censor	A state, or vector of states, which indicates censoring. See msm.
censor.states	
	Specifies the underlying states which censored observations can represent. See
	msm.

#### **Details**

Suppose we want a crude estimate of the transition intensity  $q_{rs}$  from state r to state s. If we observe  $n_{rs}$  transitions from state r to state s, and a total of  $n_r$  transitions from state r, then  $q_{rs}/q_{rr}$  can be estimated by  $n_{rs}/n_r$ . Then, given a total of  $T_r$  years spent in state r, the mean sojourn time  $1/q_{rr}$  can be estimated as  $T_r/n_r$ . Thus,  $n_{rs}/T_r$  is a crude estimate of  $q_{rs}$ .

If the data do represent the exact transition times of the Markov process, then these are the exact maximum likelihood estimates.

Observed transitions which are incompatible with the given qmatrix are ignored. Censored states are ignored.

## Value

The estimated transition intensity matrix. This can be used as the qmatrix argument to msm.

## deltamethod

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

# See Also

statetable.msm

# Examples

```
data(heart)
twoway4.q <- rbind(c(-0.5, 0.25, 0, 0.25), c(0.166, -0.498, 0.166, 0.166),
c(0, 0.25, -0.5, 0.25), c(0, 0, 0, 0))
statetable.msm(state, PTNUM, data=heart)
crudeinits.msm(state ~ years, PTNUM, data=heart, qmatrix=twoway4.q)</pre>
```

```
deltamethod
```

The delta method

## Description

Delta method for approximating the standard error of a transformation g(X) of a random variable  $X = (x_1, x_2, \ldots)$ , given estimates of the mean and covariance matrix of X.

## Usage

deltamethod(g, mean, cov, ses=TRUE)

## Arguments

g	A formula representing the transformation. The variables must be labelled $x_1$ , $x_2$ , For example,	
	~ 1 / (x1 + x2)	
	If the transformation returns a vector, then a list of formulae representing $(g_1, g_2,)$ can be provided, for example	
	list( ~ x1 + x2, ~ x1 / (x1 + x2) )	
mean	The estimated mean of $X$	
COV	The estimated covariance matrix of X	
ses	If TRUE, then the standard errors of $g_1(X), g_2(X), \ldots$ are returned. Otherwise the covariance matrix of $g(X)$ is returned.	

#### Details

The delta method expands a differentiable function of a random variable about its mean, usually with a first-order Taylor approximation, and then takes the variance. For example, an approximation to the covariance matrix of g(X) is given by

$$Cov(g(X)) = g'(\mu)Cov(X)[g'(\mu)]^T$$

where  $\mu$  is an estimate of the mean of X.

A limitation of this function is that variables created by the user are not visible within the formula g. To work around this, it is necessary to build the formula as a string, using functions such as sprintf, then to convert the string to a formula using as.formula. See the example below.

## Value

A vector containing the standard errors of  $g_1(X), g_2(X), \ldots$  or a matrix containing the covariance of g(X).

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

#### References

Oehlert, G. W. A note on the delta method. American Statistician 46(1), 1992

## Examples

```
## Simple linear regression, E(y) = alpha + beta x
x <- 1:100
y <- rnorm(100, 4*x, 5)
toy.lm <- lm(y \sim x)
estmean <- coef(toy.lm)</pre>
estvar <- summary(toy.lm)$cov.unscaled * summary(toy.lm)$sigma^2
## Estimate of (1 / (alphahat + betahat))
1 / (estmean[1] + estmean[2])
## Approximate standard error
deltamethod (~ 1 / (x1 + x2), estmean, estvar)
## We have a variable z we would like to use within the formula.
z <- 1
## deltamethod (~ z / (x1 + x2), estmean, estvar) will not work.
## Instead, build up the formula as a string, and convert to a formula.
form <- sprintf("~ %f / (x1 + x2)", z)
form
deltamethod(as.formula(form), estmean, estvar)
```

ematrix.msm

## Description

Extract the estimated misclassification probability matrix, and corresponding confidence intervals, from a fitted multi-state model at a given set of covariate values.

#### Usage

ematrix.msm(x, covariates="mean", cl=0.95)

## Arguments

Х	A fitted multi-state model, as returned by msm
covariates	The covariate values for which to estimate the misclassification probability ma- trix. This can either be:
	the string "mean", denoting the means of the covariates in the data (this is the default),
	the number 0, indicating that all the covariates should be set to zero,
	or a list of values, with optional names. For example
	list (60, 1)
	where the order of the list follows the order of the covariates originally given in
	the model formula, or a named list,
	list (age = $60$ , sex = 1)
cl	Width of the symmetric confidence interval to present. Defaults to 0.95.

## Details

Misclassification probabilities and covariate effects are estimated on the logit scale by msm. A covariance matrix is estimated from the Hessian of the maximised log-likelihood. From these, the delta method is used to obtain standard errors of the probabilities on the natural scale at arbitrary covariate values. Confidence intervals are estimated by assuming normality on the logit scale.

## Value

A list with components:

estimate	Estimated misclassification probability matrix.
SE	Corresponding approximate standard errors.
L	Lower confidence limits.
U	Upper confidence limits.

The default print method for objects returned by ematrix.msm presents estimates and confidence limits. To present estimates and standard errors, do something like

ematrix.msm(x)[c("estimates", "SE")]

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

qmatrix.msm

fev

FEV1 measurements from lung transplant recipients

## Description

A series of measurements of the forced expiratory volume in one second (FEV1) from lung transplant recipients, from six months onwards after their transplant.

#### Usage

data(fev)

#### Format

A data frame containing 5896 rows. There are 204 patients, the rows are grouped by patient number and ordered by days after transplant. Each row represents an examination and containing an additional covariate.

ptnum	(numeric)	Patient identification number.
days	(numeric)	Examination time (days after transplant).
fev	(numeric)	Percentage of baseline FEV1. A code of 999 indicates the patient's date of death.
acute	(numeric)	0/1 indicator for whether the patient suffered an acute infection or rejection
		within 14 days of the visit.

## Details

A baseline "normal" FEV1 for each individual is calculated using measurements from the first six months after transplant. After six months, as presented in this dataset, FEV1 is expressed as a percentage of the baseline value.

FEV1 is monitored to diagnose bronchiolitis obliterans syndrome (BOS), a long-term lung function decline, thought to be a form of chronic rejection. Acute rejections and infections also affect the lung function in the short term.

## hazard.msm

## Source

Papworth Hospital, U.K.

#### References

Jackson, C.H. and Sharples, L.D. Hidden Markov models for the onset and progression of bronchiolitis obliterans syndrome in lung transplant recipients *Statistics in Medicine*, 21(1): 113–128 (2002).

hazard.msm	Calculate tables of hazard ratios for covariates on transition intensi-
	ties

## Description

Hazard ratios are computed by exponentiating the estimated covariate effects on the log-transition intensities. This function is called by summary.msm.

#### Usage

hazard.msm(x, hazard.scale = 1, cl = 0.95)

## Arguments

Х	Output from msm representing a fitted multi-state model.
hazard.scale	Vector with same elements as number of covariates on transition rates. Corresponds to the increase in each covariate used to calculate its hazard ratio. Defaults to all 1.
cl	Width of the symmetric confidence interval to present. Defaults to 0.95.

#### Value

A list of tables containing hazard ratio estimates, one table for each covariate. Each table has three columns, containing the hazard ratio, and an approximate upper and lower confidence limit respectively (assuming normality on the log scale), for each Markov chain transition intensity.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

#### See Also

msm, summary.msm, odds.msm

heart

#### Description

A series of approximately yearly angiographic examinations of heart transplant recipients. The state at each time is a grade of cardiac allograft vasculopathy (CAV), a deterioration of the arterial walls.

#### Usage

data(heart)

## Format

A data frame containing 2846 rows. There are 622 patients, the rows are grouped by patient number and ordered by years after transplant, with each row representing an examination and containing additional covariates.

PTNUM	(numeric)	Patient identification number
age	(numeric)	Recipient age at examination (years)
years	(numeric)	Examination time (years after transplant)
dage	(numeric)	Age of heart donor (years)
sex	(character)	sex (0=male, 1=female)
pdiag	(character)	Primary diagnosis (reason for transplant)
		IHD=ischaemic heart disease, IDC=idiopathic dilated cardiomyopathy.
cumrej	(numeric)	Cumulative number of acute rejection episodes
state	(numeric)	State at the examination.
		State 1 represents no CAV, state 2 is mild/moderate CAV
		and state 3 is severe CAV. State 4 indicates death.

## Source

Papworth Hospital, U.K.

#### References

Sharples, L.D. and Jackson, C.H. and Parameshwar, J. and Wallwork, J. and Large, S.R. (2003). Diagnostic accuracy of coronary angiopathy and risk factors for post-heart-transplant cardiac allograft vasculopathy. Transplantation 76(4):679-82

hmm-dists

#### hmm-dists

## Description

These functions are used to specify the distribution of the response conditionally on the underlying state in a hidden Markov model. A list of these function calls, with one component for each state, should be used for the hmodel argument to msm. The initial values for the parameters of the distribution should be given as arguments.

## Usage

```
hmmCat(prob, basecat)
hmmIdent(x)
hmmUnif(lower, upper)
hmmNorm(mean, sd)
hmmLNorm(meanlog, sdlog)
hmmExp(rate)
hmmGamma(shape, rate)
hmmWeibull(shape, scale)
hmmPois(rate)
hmmBinom(size, prob)
hmmTNorm(mean, sd, lower, upper)
hmmMETNorm(mean, sd, lower, upper, sderr, meanerr=0)
hmmMEUnif(lower, upper, sderr, meanerr=0)
hmmNBinom(disp, prob)
```

## Arguments

hmmCat represents a categorical response distribution on the set 1, 2, ..., length (prob). The Markov model with misclassification is an example of this type of model. The categories in this case are (some subset of) the observed states.

The hmmIdent distribution is used for underlying states which are observed exactly without error.

hmmUnif, hmmNorm, hmmLNorm, hmmExp, hmmGamma, hmmWeibull, hmmPois, hmmBinom, hmmTNorm and hmmNBinom represent Uniform, Normal, log-Normal, exponential, Gamma, Weibull, Poisson, Binomial, truncated Normal and negative binomial distributions, respectively, with parameterisations the same as the default parameterisations in the corresponding base R distribution functions.

The hmmMETNorm and hmmMEUnif distributions are truncated Normal and Uniform distributions, but with additional Normal measurement error on the response. These are generalisations of the distributions proposed by Satten and Longini (1994) for modelling the progression of CD4 cell counts in monitoring HIV disease. See medists for density, distribution, quantile and random generation functions for these distributions. See also tnorm for density, distribution, quantile and random generation functions for the truncated Normal distribution.

prob	(hmmCat) Vector of probabilities of observing category 1, 2,, length (prob) respectively. Or the probability governing a binomial or negative binomial distribution.
basecat	(hmmCat) Category which is considered to be the "baseline", so that during estimation, the probabilities are parameterised as probabilities relative to this baseline category. By default, the category with the greatest probability is used as the baseline.

Х	(hmmIdent) Code in the data which denotes the exactly-observed state.
mean	(hmmNorm, hmmLNorm, hmmTNorm) Mean defining a Normal, or truncated Normal distribution.
sd	(hmmNorm, hmmLNorm, hmmTNorm) Standard deviation defining a Normal, or truncated Normal distribution.
meanlog	(hmmNorm, hmmLNorm, hmmTNorm) Mean on the log scale, for a log Normal distribution.
sdlog	(hmmNorm, hmmLNorm, hmmTNorm) Standard deviation on the log scale, for a log Normal distribution.
rate	(hmmPois, hmmExp, hmmGamma) Rate of a Poisson, Exponential or Gamma distribution (see dpois, dexp, dgamma).
shape	(hmmPois, hmmExp, hmmGamma) Shape parameter of a Gamma or Weibull distribution (see dgamma, dweibull).
scale	(hmmGamma) Shape parameter of a Gamma distribution (see dgamma).
size	Order of a Binomial distribution (see dbinom).
disp	Dispersion parameter of a negative binomial distribution, also called size or order. (see dnbinom).
lower	(hmmUnif, hmmTNorm, hmmMEUnif) Lower limit for an Uniform or trun- cated Normal distribution.
upper	(hmmUnif, hmmTNorm, hmmMEUnif) Upper limit for an Uniform or trun- cated Normal distribution.
sderr	(hmmMETNorm, hmmUnif) Standard deviation of the Normal measurement error distribution.
meanerr	(hmmMETNorm, hmmUnif) Additional shift in the measurement error, fixed to 0 by default. This may be modelled in terms of covariates.

## Details

See the PDF manual 'msm-manual.pdf' in the 'doc' subdirectory for algebraic definitions of all these distributions.

Parameters which can be modelled in terms of covariates, on the scale of a link function, are as follows.

LINK FUNCTION
identity
identity
log
log
identity
logit

Parameters basecat, lower, upper, size, meanerr are fixed at their initial values. All other parameters are estimated while fitting the hidden Markov model, unless the appropriate fixedpars argument is supplied to msm.

#### logLik.msm

For categorical response distributions (hmmCat) the outcome probabilities initialized to zero are fixed at zero, and the probability corresponding to basecat is fixed to one minus the sum of the remaining probabilities. These remaining probabilities are estimated, and can be modelled in terms of covariates.

## Value

Each function returns an object of class hmodel, which is a list containing information about the model. The only component which may be useful to end users is r, a function of one argument n which returns a random sample of size n from the given distribution.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

#### References

Satten, G.A. and Longini, I.M. Markov chains with measurement error: estimating the 'true' course of a marker of the progression of human immunodeficiency virus disease (with discussion) *Applied Statistics* 45(3): 275-309 (1996).

Jackson, C.H. and Sharples, L.D. Hidden Markov models for the onset and progression of bronchiolitis obliterans syndrome in lung transplant recipients *Statistics in Medicine*, 21(1): 113–128 (2002).

Jackson, C.H., Sharples, L.D., Thompson, S.G. and Duffy, S.W. and Couto, E. Multi-state Markov models for disease progression with classification error. *The Statistician*, 52(2): 193–209 (2003).

## See Also

msm

logLik.msm

Extract model log-likelihood

#### Description

Extract the log-likelihood and the number of parameters of a model fitted with msm.

## Usage

```
## S3 method for class 'msm':
logLik(object, ...)
```

#### Arguments

object	A fitted multi-state model object, as returned by msm.
	(unused) further arguments passed to or from other methods.

#### Value

The minus log-likelihood of the model represented by 'object' evaluated at the maximum likelihood estimates.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

#### See Also

msm

medists

Measurement error distributions

#### Description

Truncated Normal and Uniform distributions, where the response is also subject to a Normally distributed measurement error.

## Usage

#### Arguments

x,q	vector of quantiles.
р	vector of probabilities.
n	number of observations. If $length(n) > 1$ , the length is taken to be the number required.
mean	vector of means.
sd	vector of standard deviations.
lower	lower truncation point.
upper	upper truncation point.
sderr	Standard deviation of measurement error distribution.
meanerr	Optional shift for the measurement error distribution.
log, log.p	logical; if TRUE, probabilities $p$ are given as $log(p)$ .
lower.tail	logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$ .

medists

#### Details

The normal distribution with measurement error has density

$$\frac{\Phi(u,\mu_2,\sigma_3) - \Phi(l,\mu_2,\sigma_3)}{\Phi(u,\mu_0,\sigma_0) - \Phi(l,\mu_0,\sigma_0)}\phi(x,\mu_0 + \mu_{\epsilon},\sigma_2)$$

where

$$egin{aligned} &\sigma_2^2 = \sigma_0^2 + \sigma_\epsilon^2, \ &\sigma_3 = \sigma_0 \sigma_\epsilon / \sigma_2, \ &\mu_2 = (x-\mu_\epsilon) \sigma_0^2 + \mu_0 \sigma_\epsilon^2 \end{aligned}$$

 $\mu_0$  is the mean of the original Normal distribution before truncation,  $\sigma_0$  is the corresponding standard deviation,

ŀ

 $\boldsymbol{u}$  is the upper truncation point,

*l* is the lower truncation point,

 $\sigma_\epsilon$  is the standard deviation of the additional measurement error,

 $\mu_{\epsilon}$  is the mean of the measurement error (usually 0).

 $\phi(x)$  is the density of the corresponding normal distribution, and

 $\Phi(x)$  is the distribution function of the corresponding normal distribution.

The uniform distribution with measurement error has density

$$(\Phi(x,\mu_{\epsilon}+l,\sigma_{\epsilon})-\Phi(x,\mu_{\epsilon}+u,\sigma_{\epsilon}))/(u-l)$$

These are calculated from the original truncated Normal or Uniform density functions  $f(.|\mu, \sigma, l, u)$  as

$$\int f(y|\mu,\sigma,l,u)\phi(x,y+\mu_{\epsilon},\sigma_{\epsilon})dy$$

If sderr and meanerr are not specified they assume the default values of 0, representing no measurement error variance, and no constant shift in the measurement error, respectively.

Therefore, for example with no other arguments, dmenorm (x), is simply equivalent to dtnorm (x), which in turn is equivalent to dnorm (x).

These distributions were used by Satten and Longini (1996) for CD4 cell counts conditionally on hidden Markov states of HIV infection, and later by Jackson and Sharples (2002) for FEV1 measurements conditionally on states of chronic lung transplant rejection.

These distribution functions are just provided for convenience, and are not optimised for numerical accuracy. To fit a hidden Markov model with these response distributions, use a hmmMETNorm or hmmMEUnif constructor. See the hmm-dists help page for further details.

## Value

dmenorm, dmeunif give the density, pmenorm, pmeunif give the distribution function, qmenorm, qmeunif give the quantile function, and rmenorm, rmeunif generate random deviates, for the Normal and Uniform versions respectively.

#### Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## References

Satten, G.A. and Longini, I.M. Markov chains with measurement error: estimating the 'true' course of a marker of the progression of human immunodeficiency virus disease (with discussion) *Applied Statistics* 45(3): 275-309 (1996)

Jackson, C.H. and Sharples, L.D. Hidden Markov models for the onset and progression of bronchiolitis obliterans syndrome in lung transplant recipients *Statistics in Medicine*, 21(1): 113–128 (2002).

## See Also

dnorm, dunif, dtnorm

#### Examples

```
## what does the distribution look like?
x <- seq(50, 90, by=1)
plot(x, dnorm(x, 70, 10), type="1", ylim=c(0,0.06)) ## standard Normal
lines(x, dtnorm(x, 70, 10, 60, 80), type="1") ## truncated Normal
## truncated Normal with small measurement error
lines(x, dmenorm(x, 70, 10, 60, 80, sderr=3), type="1")
```

msm

Multi-state Markov and hidden Markov models in continuous time

#### Description

Fit a continuous-time Markov or hidden Markov multi-state model by maximum likelihood. Observations of the process can be made at arbitrary times, or the exact times of transition between states can be known. Covariates can be fitted to the Markov chain transition intensities or to the hidden Markov observation process.

## Usage

```
msm ( formula, subject=NULL, data = list(), qmatrix, gen.inits = FALSE,
ematrix=NULL, hmodel=NULL, obstype=NULL,
covariates = NULL, covinits = NULL, constraint = NULL,
misccovariates = NULL, misccovinits = NULL, miscconstraint = NULL,
hcovariates = NULL, hcovinits = NULL, hconstraint = NULL,
qconstraint=NULL, econstraint=NULL, initprobs = NULL, est.initprobs=FALSE,
death = FALSE, exacttimes = FALSE, censor=NULL,
censor.states=NULL, cl = 0.95, fixedpars = NULL, center=TRUE,
opt.method=c("optim", "nlm"), hessian=TRUE, use.deriv=FALSE,
deriv.test=FALSE, analyticp=TRUE, ...)
```

## msm

# Arguments

formula	A formula giving the vectors containing the observed states and the correspond- ing observation times. For example,
	state ~ time
	Observed states should be in the set 1, $\ldots$ , n, where n is the number of states.
subject	Vector of subject identification numbers for the data specified by formula. If missing, then all observations are assumed to be on the same subject. These must be sorted so that all observations on the same subject are adjacent.
data	Optional data frame in which to interpret the variables supplied in formula, subject, covariates, misccovariates, hcovariates and obstype.
qmatrix	Initial transition intensity matrix of the Markov chain. If an instantaneous tran- sition is not allowed from state $r$ to state $s$ , then qmatrix should have $(r, s)$ entry 0, otherwise it should be non-zero. Any diagonal entry of qmatrix is ignored, as it is constrained to be equal to minus the sum of the rest of the row. For example,
	rbind( c( 0, 0.1, 0.01 ), c( 0.1, 0, 0.2 ), c( 0, 0, 0 ) )
	represents a 'health - disease - death' model, with transition intensities 0.1 from health to disease, 0.01 from health to death, 0.1 from disease to health, and 0.2 from disease to death. The initial intensities given here are with any covariates set to their means in the data (or set to zero, if center = FALSE).
gen.inits	If TRUE, then initial values for the transition intensities are estimated by assuming that the data represent the exact transition times of the process. The non-zero entries of the supplied qmatrix are assumed to indicate the allowed transitions of the model.
ematrix	If misclassification between states is to be modelled, this should be a matrix of initial values for the misclassification probabilities. The rows represent underlying states, and the columns represent observed states. If an observation of state $s$ is not possible when the subject occupies underlying state $r$ , then ematrix should have $(r, s)$ entry 0. Otherwise ematrix should have $(r, s)$ entry corresponding to the probability of observing $s$ conditionally on occupying true state $r$ . The diagonal of ematrix is ignored, as rows are constrained to sum to 1. For example,
	rbind( c( 0, 0.1, 0 ), c( 0.1, 0, 0.1 ), c( 0, 0.1, 0 ) )
	represents a model in which misclassifications are only permitted between adja- cent states.
	For an alternative way of specifying misclassification models, see hmodel.
hmodel	Specification of the hidden Markov model. This should be a list of return values from the constructor functions described in the hmm-dists help page. Each

element of the list corresponds to the outcome model conditionally on the corresponding underlying state.

For example, consider a three-state hidden Markov model. Suppose the observations in underlying state 1 are generated from a Normal distribution with mean 100 and standard deviation 16, while observations in underlying state 2 are Normal with mean 54 and standard deviation 18. Observations in state 3, representing death, are exactly observed, and coded as 999 in the data. This model is specified as

hmodel = list(hmmNorm(mean=100, sd=16), hmmNorm(mean=54, sd=18), hmmIdent(999))

The mean and standard deviation parameters are estimated starting from these initial values. See the hmm-dists help page for details of the constructor functions for each available distribution.

A misclassification model, that is, a hidden Markov model where the outcomes are misclassified observations of the underlying states, can either be specified using a list of hmmCat objects, or by using an ematrix as in previous versions of **msm**.

For example,

ematrix = rbind( c( 0, 0.1, 0, 0 ), c( 0.1, 0, 0.1, 0
), c( 0, 0.1, 0, 0), c( 0, 0, 0, 0) )

is equivalent to

hmodel =	= list(	hmmCat(prob=c(0.	9, 0.1,	, 0,	0)),	<pre>hmmCat(prob=c(0.1,</pre>
0.8, 0.1	L, O)),	hmmCat(prob=c(0,	0.1, 0	).9,	0)),	hmmIdent())

obstype	A vector specifying the observation scheme for each row of the data. This can
	be included in the data frame data along with the state, time, subject IDs and
	covariates. Its elements should be either 1, 2 or 3, meaning as follows:

- 1 An observation of the process at an arbitrary time (a "snapshot" of the process)
- **2** An exact transition time, with the state at the previous observation retained until the current observation.
- **3** An exact transition time, but the state at the instant before entering this state is unknown. A common example is death times in studies of chronic diseases.

If obstype is not specified, this defaults to all 1. If obstype is a single number, all observations are assumed to be of this type.

This is a generalisation of the death and exact times arguments to allow different schemes per observation.

exacttimes=TRUE specifies that all observations are of obstype 2.

death = death.states specifies that all observations of death.states are of type 3. death = TRUE specifies that all observations in the final absorbing state are of type 3.

covariates Formula representing the covariates on the transition intensities via a log-linear model. For example,

~ age + sex + treatment

covinits	Initial values for log-linear effects of covariates on the transition intensities. This should be a named list with each element corresponding to a covariate. A single element contains the initial values for that covariate on each transition intensity, reading across the rows in order. For a pair of effects constrained to be equal, the initial value for the first of the two effects is used.
	For example, for a model with the above qmatrix and age and sex covariates, the following initialises all covariate effects to zero apart from the age effect on the 2-1 transition, and the sex effect on the 1-3 transition. $covinits = list(sex=c(0, 0, 0.1, 0), age=c(0, 0.1, 0, 0))$
	For factor covariates, name each level by concatenating the name of the co- variate with the level name, quoting if necessary. For example, for a covariate agegroup with three levels 0-15, 15-60, 60-, use something like
	<pre>covinits = list("agegroup15-60"=c(0, 0.1, 0, 0), "agegroup60- "=c(0.1, 0.1, 0, 0))</pre>
	If not specified or wrongly specified, initial values are assumed to be zero.
constraint	A list of one vector for each named covariate. The vector indicates which co- variate effects on intensities are constrained to be equal. Take, for example, a model with five transition intensities and two covariates. Specifying

```
constraint = list (age = c(1, 1, 1, 2, 2), treatment = c(1, 2, 3, 4, 5))
```

constrains the effect of age to be equal for the first three intensities, and equal for the fourth and fifth. The effect of treatment is assumed to be different for each intensity. Any vector of increasing numbers can be used as indicators. The intensity parameters are assumed to be ordered by reading across the rows of the transition matrix, starting at the first row, ignoring the diagonals.

For categorical covariates, defined using factor (covname), specify constraints as follows:

```
list(..., covnameVALUE1 = c(...), covnameVALUE2 = c(...),
...)
```

where VALUE1, VALUE2, ... are the levels of the factor. Make sure the contrasts option is set appropriately, for example, the default

options(contrasts=c(contr.treatment, contr.poly))

sets the first (baseline) level of unordered factors to zero.

To assume no covariate effect on a certain transition, set its initial value to zero and use the fixedpars argument to fix it during the optimisation.

#### misccovariates

A formula representing the covariates on the misclassification probabilities, analogously to covariates. Only used if the model is specified using ematrix, rather than hmodel.

misccovinits Initial values for the covariates on the misclassification probabilities, defined in the same way as covinits. Only used if the model is specified using ematrix.

#### miscconstraint

A list of one vector for each named covariate on misclassification probabilities. The vector indicates which covariate effects on misclassification probabilities are constrained to be equal, analogously to constraint. Only used if the model is specified using ematrix.

- hcovariates List of formulae the same length as hmodel, defining any covariates governing the hidden Markov outcome models. The covariates operate on a suitably linktransformed linear scale, for example, log scale for a Poisson outcome model. If there are no covariates for a certain hidden state, then insert a NULL in the corresponding place in the list. For example, hcovariates = list (~acute + age, ~acute, NULL).
- hcovinits Initial values for the hidden Markov model covariates. A list of the same length as hcovariates. Each element is a vector with initial values for each covariate on that state. For example, the above hcovariates can be initialised with hcovariates = list(c(-8, 0), -8, NULL). Initial values must be given for all or no covariates, if none are given these are all set to zero. The initial value given in the hmodel constructor function for the corresponding baseline parameter is interpreted as the value of that parameter with any covariates fixed to their means in the data.
- hconstraint A named list. Each element is a vector of constraints on the named hidden Markov model parameter. The vector has length equal to the number of times that class of parameter appears in the whole model.

For example consider the three-state hidden Markov model described above, with normally-distributed outcomes for states 1 and 2. To constrain the outcome variance to be equal for states 1 and 2, and to also constrain the effect of acute on the outcome mean to be equal for states 1 and 2, specify

hconstraint = list(sd = c(1,1), acute=c(1,1))

qconstraint A vector of indicators specifying which baseline transition intensities are equal. For example,

qconstraint = c(1,2,3,3)

constrains the third and fourth intensities to be equal, in a model with four allowed instantaneous transitions.

- econstraint A similar vector of indicators specifying which baseline misclassification probabilities are constrained to be equal. Only used if the model is specified using ematrix, rather than hmodel.
- initprobs Currently only used in hidden Markov models. Vector of assumed underlying state occupancy probabilities at each individual's first observation. If these are estimated (see est.initprobs), then this defaults to equal probability for each state. Otherwise this defaults to c(1, rep(0, nstates-1)), that is, in state 1 with a probability of 1. Scaled to sum to 1 if necessary.

est.initprobs

If TRUE, then the underlying state occupancy probabilities at the first observation will be estimated, starting from initial values taken from the initprobs argument. Be warned that if any of these initial values are 0 or 1, then optim will give an "non-finite value" error. To fix any of these probabilities during the estimation, e.g. at values of 0 or 1, then use an appropriate fixedpars argument. Note that the free parameters during this estimation excludes the state 1 occupancy probability, which is fixed at 1 minus the sum of the other probabilities.

death Vector of indices of the death states. A death state is an absorbing state whose time of entry is known exactly, but the individual is assumed to be in an unknown transient state ("alive") at the previous instant. This is the usual situation for times of death in chronic disease monitoring data. For example, if you specify death = c(4, 5) then states 4 and 5 are assumed to be death states. death = TRUE indicates that the final state is a death state, and death = FALSE (the default) indicates that there is no death state. See the obstype argument. censor A state, or vector of states, which indicates censoring. Censoring means that the observed state is known only to be one of a particular set of states. For example, censor=999 indicates that all observations of 999 in the vector of observed states denote censoring times. By default, this means that the true state could have been anything other than an absorbing state. To specify corresponding true states explicitly, use a censor.states argument. censor.states Specifies the underlying states which censored observations can represent. If censor is a single number (the default) this can be a vector, or a list with one element. If censor is a vector with more than one element, this should be a list, with each element a vector corresponding to the equivalent element of censor. For example censor = c(99, 999), censor.states = list(c(2,3), c(3,4)) means that observations coded 99 represent either state 2 or state 3, while observations coded 999 are really either state 3 or state 4. By default, the transitions of the Markov process are assumed to take place exacttimes at unknown occasions in between the observation times. If exacttimes is set to TRUE, then all observation times are assumed to represent the exact and complete times of transition of the process. This is equivalent to every row of the data having obstype = 2. See the obstype argument. cl Width of symmetric confidence intervals for maximum likelihood estimates, by default 0.95. fixedpars Vector of indices of parameters whose values will be fixed at their initial values during the optimisation. These are given in the order: transition intensities (reading across rows of the transition matrix), covariates on intensities (ordered by intensities within covariates), hidden Markov model parameters (ordered by parameters within states), hidden Markov model covariate parameters (ordered by covariates within parameters within states), initial state occupancy probabilities (excluding the first probability, which is fixed at one minus the sum of the others). For covariates on misclassification probabilities, this is a change from version 0.4 in the parameter ordering. Previously these were ordered by misclassification probabilities within covariates. This can be useful for profiling likelihoods, and building complex models stage by stage. To fix all parameters, specify fixedpars = TRUE. If TRUE (the default) then covariates are centered at their means during the center maximum likelihood estimation. This usually improves convergence.

opt.method	Quoted name of the R function to perform minimisation of the minus twice log likelihood. Either "optim" or "nlm". optim is the default.
hessian	If TRUE (the default) then the Hessian matrix is computed at the maximum likelihood estimates, to obtain standard errors and confidence intervals.
use.deriv	If TRUE then analytic first derivatives are used in the optimisation of the likeli- hood, when an appropriate quasi-Newton optimisation method, such as BFGS, is being used. Note that the default for optim is a Nelder-Mead method which cannot use derivatives. However, these derivatives, if supplied, are always used to calculate the Hessian.
deriv.test	If TRUE, then analytic and numeric derivatives are computed and compared at the initial values, and no optimisation is performed.
analyticp	By default, the likelihood for certain simpler 3, 4 and 5 state models is calculated using an analytic expression for the transition probability (P) matrix. To revert to the original method of using the matrix exponential, specify analyticp=FALSE See the PDF manual for a list of the models for which analytic P matrices are implemented.
	Optional arguments to the general-purpose R optimisation routines, optim or nlm. Useful options for optim include method="BFGS" for using a quasi-Newton optimisation algorithm, which can often be faster than the default Nelder-Mead. If the optimisation fails to converge, consider normalising the problem using, for example, control=list(fnscale = 2500), for ex- ample, replacing 2500 by a number of the order of magnitude of the likelihood. If 'false' convergence is reported and the standard errors cannot be calculated due to a non-positive-definite Hessian, then consider tightening the tolerance cri- teria for convergence. If the optimisation takes a long time, intermediate steps can be printed using the trace argument of the control list. See optim for details.

#### Details

For full details about the methodology behind the **msm** package, refer to the PDF manual 'msm-manual.pdf' in the 'doc' subdirectory of the package. This includes a tutorial in the typical use of **msm**.

Users upgrading from versions of **msm** less than 0.5 will need to change some of their model fitting syntax. In particular, initial values are now specified in the qmatrix and covinits arguments instead of inits, and qmatrix is no longer a matrix of 0/1 indicators. See the appendix to the PDF manual or the NEWS file in the top-level installation directory for a full list of changes.

For simple multi-state Markov models, the likelihood is calculated in terms of the transition intensity matrix Q. When the data consist of observations of the Markov process at arbitrary times, the exact transition times are not known. Then the likelihood is calculated using the transition probability matrix  $P(t) = \exp(tQ)$ , where exp is the matrix exponential. If state *i* is observed at time *t* and state *j* is observed at time *u*, then the contribution to the likelihood from this pair of observations is the *i*, *j* element of P(u - t). See, for example, Kalbfleisch and Lawless (1985), Kay (1986), or Gentleman *et al.* (1994).

For hidden Markov models, the likelihood for an individual with k observations is calculated directly by summing over the unknown state at each time, producing a product of k matrices. The

msm

calculation is a generalisation of the method described by Satten and Longini (1996), and also by Jackson and Sharples (2002), and Jackson *et al.* (2003).

There must be enough information in the data on each state to estimate each transition rate, otherwise the likelihood will be flat and the maximum will not be found. It may be appropriate to reduce the number of states in the model, or reduce the number of covariate effects, to ensure convergence. Hidden Markov models are particularly susceptible to non-identifiability, especially when combined with a complex transition matrix.

Choosing an appropriate set of initial values for the optimisation can also be important. For flat likelihoods, 'informative' initial values will often be required.

### Value

A list of class msm, with components:

call	The original call to msm.	
Qmatrices	A list of matrices. The first component, labelled logbaseline, is a matrix containing the estimated transition intensities on the log scale with any covariates fixed at their means in the data. Each remaining component is a matrix giving the linear effects of the labelled covariate on the matrix of log intensities. To extract an estimated intensity matrix on the natural scale, at an arbitrary combination of covariate values, use the function <code>qmatrix.msm</code> .	
QmatricesSE	The standard error matrices corresponding to Qmatrices.	
QmatricesL,Qr	natricesU	
	Corresponding lower and upper symmetric confidence limits, of width 0.95 unless specified otherwise by the cl argument.	
Ematrices	A list of matrices. The first component, labelled logitbaseline, is the es- timated misclassification probability matrix with any covariates fixed at their means in the data. Each remaining component is a matrix giving the linear effects of the labelled covariate on the matrix of logit misclassification prob- abilities. To extract an estimated misclassification probability matrix on the natural scale, at an arbitrary combination of covariate values, use the function ematrix.msm.	
EmatricesSE	The standard error matrices corresponding to Ematrices.	
EmatricesL, EmatricesU		
	Corresponding lower and upper symmetric confidence limits, of width 0.95 unless specified otherwise by the cl argument.	
sojourn	A list with components:	
	mean = estimated mean sojourn times in the transient states, with covariates fixed at their means.	
	se = corresponding standard errors.	
minus2loglik	Minus twice the maximised log-likelihood.	
estimates	Vector of untransformed maximum likelihood estimates returned from optim. Transition intensities are on the log scale and misclassification probabilities are the logit scale.	
estimates.t	Vector of transformed maximum likelihood estimates with intensities and prob- abilities on their natural scales.	

fixedpars	Indices of ${\tt estimates}$ which were fixed during the maximum likelihood estimation.
covmat	Covariance matrix corresponding to estimates.
ci	Matrix of confidence intervals corresponding to estimates
opt	Return value from optim or nlm, giving information about the results of the optimisation.
foundse	Logical value indicating whether the Hessian was positive-definite at the sup- posed maximum of the likelihood. If not, the covariance matrix of the parame- ters is unavailable. In these cases the optimisation has probably not converged to a maximum.
data	A list of constants and vectors giving the data, for use in post-processing.
qmodel	A list of objects specifying the model for transition intensities, for use in post-processing.
emodel	A list of objects specifying the model for misclassification.
qcmodel	A list of objects specifying the model for covariates on the transition intensities.
ecmodel	A list of objects specifying the model for covariates on misclassification probabilities.
hmodel	A list of class "hmodel", containing objects specifying the hidden Markov model. Estimates of "baseline" location parameters are presented with any covariates fixed to their means in the data.
cmodel	A list of objects specifying any model for censoring.

Printing a msm object by typing the object's name at the command line implicitly invokes print.msm. This formats and prints the important information in the model fit. This includes the fitted transition intensity matrix, matrices containing covariate effects on intensities, and mean sojourn times from a fitted msm model. When there is a hidden Markov model, the chief information in the hmodel component is also formatted and printed. This includes estimates and confidence intervals for each parameter.

To extract summary information from the fitted model, it is recommended to use the more flexible extractor functions, such as qmatrix.msm, pmatrix.msm, sojourn.msm, instead of directly reading from list components of msm objects.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

#### References

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Satten, G.A. and Longini, I.M. Markov chains with measurement error: estimating the 'true' course of a marker of the progression of human immunodeficiency virus disease (with discussion) *Applied Statistics* 45(3): 275-309 (1996)

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Jackson, C.H., Sharples, L.D., Thompson, S.G. and Duffy, S.W. and Couto, E. Multi-state Markov models for disease progression with classification error. *The Statistician*, 52(2): 193–209 (2003)

#### See Also

simmulti.msm, plot.msm, summary.msm, qmatrix.msm, pmatrix.msm, sojourn.msm.

## Examples

```
### Heart transplant data
### For further details and background to this example, see
### the PDF manual in the doc directory.
data(heart)
print(heart[1:10,])
twoway4.q <- rbind(c(-0.5, 0.25, 0, 0.25), c(0.166, -0.498, 0.166, 0.166),
c(0, 0.25, -0.5, 0.25), c(0, 0, 0, 0))
statetable.msm(state, PTNUM, data=heart)
crudeinits.msm(state ~ years, PTNUM, data=heart, qmatrix=twoway4.q)
heart.msm <- msm( state ~ years, subject=PTNUM, data = heart,
                 qmatrix = twoway4.q, death = 4,
                 control = list ( trace = 2, REPORT = 1 ) )
heart.msm
qmatrix.msm(heart.msm)
pmatrix.msm(heart.msm, t=10)
sojourn.msm(heart.msm)
```

odds.msm	Calculate tables	of odds	ratios for	covariates	on	misclassification
	probabilities					

#### Description

Odds ratios are computed by exponentiating the estimated covariate effects on the logit-misclassification probabilities.

#### Usage

```
odds.msm(x, odds.scale = 1, cl = 0.95)
```

## Arguments

Х	Output from msm representing a fitted multi-state model.
odds.scale	Vector with same elements as number of covariates on misclassification proba- bilities. Corresponds to the increase in each covariate used to calculate its odds ratio. Defaults to all 1.
cl	Width of the symmetric confidence interval to present. Defaults to 0.95.

## Value

A list of tables containing odds ratio estimates, one table for each covariate. Each table has three columns, containing the odds ratio, and an approximate upper 95% and lower 95% confidence limit respectively (assuming normality on the log scale), for each misclassification probability.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

msm, hazard.msm

pexp

Exponential distribution with piecewise-constant rate

## Description

Density, distribution function, quantile function and random generation for a generalisation of the exponential distribution, in which the rate changes at a series of times.

## Usage

```
dpexp(x, rate=1, t=0, log = FALSE)
ppexp(q, rate=1, t=0, lower.tail = TRUE, log.p = FALSE)
qpexp(p, rate=1, t=0, lower.tail = TRUE, log.p = FALSE)
rpexp(n, rate=1, t=0)
```

## Arguments

x,q	vector of quantiles.
р	vector of probabilities.
n	number of observations. If length(n) $>$ 1, the length is taken to be the number required.
rate	vector of rates.
t	vector of the same length as rate, giving the times at which the rate changes. The first element of t should be 0, and t should be in increasing order.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$ .

#### plot.msm

#### **Details**

Consider the exponential distribution with rates  $r_1, \ldots, r_n$  changing at times  $t_1, \ldots, t_n$ , with  $t_1 = 0$ . Suppose  $t_k$  is the maximum  $t_i$  such that  $t_i < x$ . The density of this distribution at x > 0 is f(x) for k = 1, and

$$\prod_{i=1}^{k} (1 - F(t_i - t_{i-1}, r_i)) f(x - t_k, r_k)$$

for k > 1.

where F() and f() are the distribution and density functions of the standard exponential distribution.

If rate is of length 1, this is just the standard exponential distribution. Therefore, for example, dpexp(x), with no other arguments, is simply equivalent to dexp(x).

Only rpexp is used in the msm package, to simulate from Markov processes with piecewiseconstant intensities depending on time-dependent covariates. These functions are merely provided for completion, and are not optimized for numerical stability.

#### Value

dpexp gives the density, ppexp gives the distribution function, qpexp gives the quantile function, and rpexp generates random deviates.

#### Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

#### See Also

dexp, sim.msm.

#### Examples

```
x <- seq(0.1, 50, by=0.1)
rate <- c(0.1, 0.2, 0.05, 0.3)
t <- c(0, 10, 20, 30)
plot(x, dexp(x, 0.1), type="l") ## standard exponential distribution
lines(x, dpexp(x, rate, t), type="l", lty=2) ## distribution with piecewise constant rate
plot(x, pexp(x, 0.1), type="l") ## standard exponential distribution
lines(x, ppexp(x, rate, t), type="l", lty=2) ## distribution with piecewise constant rate</pre>
```

plot.msm

Plots of multi-state models

#### Description

This produces a plot of the expected probability of survival against time, from each transient state. Survival is defined as not entering an absorbing state.

# Usage

```
## S3 method for class 'msm':
plot(x, from, to, range, covariates, legend.pos, ...)
```

# Arguments

Х	Output from msm, representing a fitted multi-state model object
from	States from which to consider survival. Defaults to the complete set of transient states.
to	Absorbing state to consider. Defaults to the highest-labelled absorbing state.
range	Vector of two elements, giving the range of times to plot for.
covariates	Covariate values for which to evaluate the expected probabilities. This can either be:
	the string "mean", denoting the means of the covariates in the data (this is the default),
	the number 0, indicating that all the covariates should be set to zero,
	or a list of values, with optional names. For example
	list (60, 1)
	where the order of the list follows the order of the covariates originally given in the model formula, or a named list,
	list (age = 60, sex = 1)
legend.pos	Vector of the $x$ and $y$ position, respectively, of the legend.
	Other arguments to the generic plot function

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

msm

pmatrix.msm Transition probability matrix

# Description

Extract the estimated transition probability matrix from a fitted multi-state model for a given time interval, at a given set of covariate values.

#### pmatrix.msm

#### Usage

```
pmatrix.msm(x, t=1, covariates="mean", ci.boot=FALSE, cl=0.95, B=500)
```

#### Arguments

Х	A fitted multi-state model, as returned by msm.		
t	The time interval to estimate the transition probabilities for, by default one unit.		
covariates	The covariate values at which to estimate the transition probabilities. This can either be:		
	the string "mean", denoting the means of the covariates in the data (this is the default),		
	the number 0, indicating that all the covariates should be set to zero,		
	or a list of values, with optional names. For example		
	list (60, 1)		
	where the order of the list follows the order of the covariates originally given in the model formula, or a named list,		
	list (age = 60, sex = 1)		
ci.boot	Calculate a bootstrap confidence interval. This is usually time-consuming, and disabled by default. See boot.msm for more details of bootstrapping in <b>msm</b> .		
cl	Width of the symmetric confidence interval		
В	Number of bootstrap replicates		

#### Details

For a continuous-time homogeneous Markov process with transition intensity matrix Q, the probability of occupying state s at time u + t conditionally on occupying state r at time u is given by the (r, s) entry of the matrix  $P(t) = \exp(tQ)$ .

For non-homogeneous processes, where covariates and hence the transition intensity matrix are time-dependent, but are piecewise-constant within the time interval [u, u+t], the function pmatrix.piecewise.msm can be used.

## Value

The matrix of estimated transition probabilities P(t) in the given time. Rows correspond to "fromstate" and columns to "to-state".

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

qmatrix.msm, pmatrix.piecewise.msm, boot.msm

```
pmatrix.piecewise.msm
```

Transition probability matrix for processes with piecewise-constant intensities

#### Description

Extract the estimated transition probability matrix from a fitted non-time-homogeneous multi-state model for a given time interval. This is a generalisation of pmatrix.msm to non-homogeneous models with time-dependent covariates.

#### Usage

```
pmatrix.piecewise.msm(x, t1, t2, times, covariates)
```

#### Arguments

X	A fitted multi-state model, as returned by msm. This should be a non-homogeneous model, whose transition intensity matrix depends on a time-dependent covariate.		
t1	The start of the time interval to estimate the transition probabilities for.		
t2	The end of the time interval to estimate the transition probabilities for.		
times	Cut points at which the transition intensity matrix changes.		
covariates	A list with number of components one greater than the length of times. Each component of the list is specified in the same way as the covariates argument to pmatrix.msm. The components correspond to the covariate values in the intervals		
	(t1, times[1]], (times[1], times[2]],, ( times[length(times)], t2]		
	(assuming that all elements of times are in the interval (t1, t2)).		

#### Details

Suppose a multi-state model has been fitted, in which the transition intensity matrix Q(x(t)) is modelled in terms of time-dependent covariates x(t). The transition probability matrix  $P(t_1, t_n)$  for the time interval  $(t_1, t_n)$  cannot be calculated from the estimated intensity matrix as  $\exp((t_n - t_1)Q)$ , because Q varies within the interval  $t_1, t_n$ . However, if the covariates are piecewise-constant, or can be approximated as piecewise-constant, then we can calculate  $P(t_1, t_n)$  by multiplying together individual matrices  $P(t_i, t_{i+1}) = \exp((t_{i+1} - t_i)Q)$ , calculated over intervals where Q is constant:

$$P(t_1, t_n) = P(t_1, t_2) P(t_2, t_3) \dots P(t_{n-1}, t_n)$$

## Value

The matrix of estimated transition probabilities P(t) for the time interval [t1, tn]. That is, the probabilities of occupying state s at time  $t_n$  conditionally on occupying state r at time  $t_1$ . Rows correspond to "from-state" and columns to "to-state".

#### prevalence.msm

#### Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

#### See Also

pmatrix.msm

#### Examples

```
## Not run:
## In a clinical study, suppose patients are given a placebo in the
## first 5 weeks, then they begin treatment 1 at 5 weeks, and
## a combination of treatments 1 and 2 from 10 weeks.
## Suppose a multi-state model x has been fitted for the patients'
## progress, with treat1 and treat2 as time dependent covariates.
## Cut points for when treatment covariate changes
times <- c(0, 5, 10)
## Indicators for which treatments are active at the three cut points
covariates <- list( list (treat1=0, treat2=0), list(treat1=1, treat2=0),
list(treat1=1, treat2=1) )
## Calculate transition probabilities from the start of the study to 15 weeks
pmatrix.piecewise.msm(x, 0, 15, times, covariates)
## End(Not run)
```

prevalence.msm Tables of observed and expected prevalences

## Description

This provides a rough indication of the goodness of fit of a multi-state model, by estimating the observed numbers of individuals occupying each state at a series of times, and comparing these with forecasts from the fitted model.

#### Usage

#### Arguments

Х	A fitted multi-state model produced by msm.
times	Series of times at which to compute the observed and expected prevalences of states.
timezero	Initial time of the Markov process. Expected values are forecasted from here. Defaults to the minimum of the observation times given in the data.

initstates	Optional vector of the same length as the number of states. Gives the numbers of individuals occupying each state at the initial time. The default is those observed in the data.	
covariates	Covariate values for which to forecast expected state occupancy. See qmatrix.ms Defaults to the mean values of the covariates in the data set.	
misccovariate	es	
	(Misclassification models only) Values of covariates on the misclassification probability matrix for which to forecast expected state occupancy. Defaults to the mean values of the covariates in the data set.	

## Details

To compute 'observed' prevalences at a time t, individuals are assumed to be in the same state as at their last observation time preceding t.

The fitted transition probability matrix is used to forecast expected prevalences from the state occupancy at the initial time. To produce the expected number in state j at time t after the start, the number of individuals under observation at time t (including those who have died, but not those lost to follow-up) is multiplied by the product of the proportion of individuals in each state at the initial time and the transition probability matrix in the time interval t. The proportion of individuals in each state at the "initial" time is estimated, if necessary, in the same way as the observed prevalences.

For misclassification models (fitted using an ematrix), this aims to assess the fit of the full model for the *observed* states. That is, the combined Markov progression model for the true states and the misclassification model. Thus, expected prevalences of true states are estimated from the assumed proportion occupying each state at the initial time using the fitted transition probabiliy matrix. The vector of expected prevalences of true states is then multiplied by the fitted misclassification probability matrix to obtain the expected prevalences of observed states.

For general hidden Markov models, the observed state is taken to be the predicted underlying state from the Viterbi algorithm (viterbi.msm). The goodness of fit of these states to the underlying Markov model is tested.

For an example of this approach, see Gentleman et al. (1994).

#### Value

A list with components:

Observed	Table of observed numbers of individuals in each state at each time
Observed	percentages
	Corresponding percentage of the individuals at risk at each time.
Expected	Table of corresponding expected numbers.
Expected	percentages
	Corresponding percentage of the individuals at risk at each time.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

0.

#### psor

#### References

Gentleman, R.C., Lawless, J.F., Lindsey, J.C. and Yan, P. Multi-state Markov models for analysing incomplete disease history data with illustrations for HIV disease. *Statistics in Medicine* (1994) 13(3): 805–821.

#### See Also

msm, summary.msm

psor

Psoriatic arthritis data

## Description

A series of observations of grades of psoriatic arthritis, as indicated by numbers of damaged joints.

#### Usage

data (psor)

#### Format

A data frame containing 806 observations, representing visits to a psoriatic arthritis (PsA) clinic from 305 patients. The rows are grouped by patient number and ordered by examination time. Each row represents an examination and contains additional covariates.

ptnum	(numeric)	Patient identification number		
months	(numeric)	Examination time in months		
state	(numeric)	Clinical state of PsA. Patients in states 1, 2, 3 and 4		
		have 0, 1 to 4, 5 to 9 and 10 or more damaged joints,		
		respectively.		
hieffusn	(numeric)	Presence of five or more effusions		
ollwsdrt	(character)	Erythrocyte sedimentation rate of less than 15 mm/h		

#### References

Gladman, D. D. and Farewell, V.T. (1999) Progression in psoriatic arthritis: role of time-varying clinical indicators. J. Rheumatol. 26(11):2409-13

## Examples

## Four-state progression-only model with high effusion and low
## sedimentation rate as covariates on the progression rates. High
## effusion is assumed to have the same effect on the 1-2, 2-3, and 3-4
## progression rates, while low sedimentation rate has the same effect
## on the 1-2 and 2-3 intensities, but a different effect on the 3-4.

data(psor)

qmatrix.msm Transition intensity matrix

#### Description

Extract the estimated transition intensity matrix, and the corresponding standard errors, from a fitted multi-state model at a given set of covariate values.

## Usage

qmatrix.msm(x, covariates="mean", sojourn=FALSE, cl=0.95)

#### Arguments

Х	A fitted multi-state model, as returned by msm
covariates	The covariate values at which to estimate the intensity matrix. This can either be:
	the string "mean", denoting the means of the covariates in the data (this is the default),
	the number 0, indicating that all the covariates should be set to zero,
	or a list of values, with optional names. For example
	list (60, 1)
	where the order of the list follows the order of the covariates originally given in the model formula, or a named list,
	list (age = $60$ , sex = 1)
sojourn	Set to TRUE if the estimated sojourn times and their standard errors should also be returned.
cl	Width of the symmetric confidence interval to present. Defaults to 0.95.

## Details

Transition intensities and covariate effects are estimated on the log scale by msm. A covariance matrix is estimated from the Hessian of the maximised log-likelihood. The delta method is used to obtain from these the standard error of the intensities on the natural scale at arbitrary covariate values. Confidence limits are calculated by assuming normality on the log scale.

38

## qratio.msm

## Value

A list with components:

estimate	Estimated transition intensity matrix.
SE	Corresponding approximate standard errors.
L	Lower confidence limits
U	Upper confidence limits

If sojourn is TRUE, extra components called sojourn and sojournSE are included, containing the estimate and standard errors, respectively, of the mean sojourn times in each transient state.

The default print method for objects returned by qmatrix.msm presents estimates and confidence limits. To present estimates and standard errors, do something like

```
qmatrix.msm(x)[c("estimates", "SE")]
```

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

pmatrix.msm, sojourn.msm, deltamethod, ematrix.msm

qratio.msm

Estimated ratio of transition intensities

## Description

Compute the estimate and approximate standard error of the ratio of two estimated transition intensities from a fitted multi-state model at a given set of covariate values.

#### Usage

```
qratio.msm(x, ind1, ind2, covariates = "mean", cl = 0.95)
```

## Arguments

Х	A fitted multi-state model, as returned by msm
ind1	Pair of numbers giving the indices in the intensity matrix of the numerator of the ratio, for example, $c(1, 2)$ .
ind2	Pair of numbers giving the indices in the intensity matrix of the denominator of the ratio, for example, $c(2, 1)$ .

covariates The covariate values at which to estimate the intensities. This can either be:

the string "mean", denoting the means of the covariates in the data (this is the default),

the number 0, indicating that all the covariates should be set to zero,

or a list of values, with optional names. For example list (60, 1) where the order of the list follows the order of the covariates originally given in the model formula, or a named list, list (age = 60, sex = 1)

Cl Width of the symmetric confidence interval to present. Defaults to 0.95.

#### Details

For example, we might want to compute the ratio of the progression rate and recovery rate for a fitted model disease.msm with a health state (state 1) and a disease state (state 2). In this case, the progression rate is the (1,2) entry of the intensity matrix, and the recovery rate is the (2,1) entry. Thus to compute this ratio with covariates set to their means, we call

qratio.msm(disease.msm, c(1,2), c(2,1)).

Standard errors are estimated by the delta method. Confidence limits are estimated by assuming normality on the log scale.

## Value

A named vector with elements estimate, se, L and U containing the estimate, standard error, lower and upper confidence limits, respectively, of the ratio of intensities.

#### Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

#### See Also

qmatrix.msm

sim.msm	Simulate one	individual	trajectory	from	а	continuous-time	Markov
	model						

## Description

Simulate one realisation from a continuous-time Markov process up to a given time.

## sim.msm

## Usage

```
sim.msm(qmatrix, maxtime, covs=NULL, beta=NULL, obstimes=0, start=1,
mintime=0)
```

# Arguments

qmatrix	The transition intensity matrix of the Markov process. The diagonal of qmatrix is ignored, and computed as appropriate so that the rows sum to zero. For example, a possible qmatrix for a three state illness-death model with recovery is: rbind(c(0, 0.1, 0.02), c(0.1, 0, 0.01), c(0, 0, 0, 0))
maxtime	Maximum time for the simulated process.
COVS	Matrix of time-dependent covariates, with one row for each observation time and one column for each covariate.
beta	Matrix of linear covariate effects on log transition intensities. The rows corre- spond to different covariates, and the columns to the transition intensities. The intensities are ordered by reading across rows of the intensity matrix, starting with the first, counting the positive off-diagonal elements of the matrix.
obstimes	Vector of times at which the covariates are observed.
start	Starting state of the process. Defaults to 1.
mintime	Starting time of the process. Defaults to 0.

## Details

The effect of time-dependent covariates on the transition intensity matrix for an individual is determined by assuming that the covariate is a step function which remains constant in between the individual's observation times.

## Value

A list with components,

states	Simulated states through which the process moves. This ends with either an absorption before obstime, or a transient state at obstime.
times	Exact times at which the process changes to the corresponding states
qmatrix	The given transition intensity matrix

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

simmulti.msm

# Examples

simmulti.msm

Simulate multiple trajectories from a multi-state Markov model with arbitrary observation times

## Description

Simulate a number of individual realisations from a multi-state Markov process. Observations of the process are made at specified arbitrary times for each individual.

#### Usage

```
simmulti.msm(data, qmatrix, covariates=NULL, death = FALSE, start,
ematrix=NULL, hmodel=NULL, hcovariates=NULL)
```

## Arguments

data	A data frame with a mandatory column named time, representing observation times. The optional column named subject, corresponds to subject identifi- cation numbers. If not given, all observations are assumed to be on the same individual. Observation times should be sorted within individuals. Other named columns of the data frame represent any covariates.
qmatrix	The transition intensity matrix of the Markov process, with any covariates set to zero. The diagonal of qmatrix is ignored, and computed as appropriate so that the rows sum to zero. For example, a possible qmatrix for a three state illness-death model with recovery is:
	rbind( c( 0, 0.1, 0.02 ), c( 0.1, 0, 0.01 ), c( 0, 0, 0 ))
covariates	List of covariate effects on log transition intensities. Each element is a vector of the effects of one covariate on all the transition intensities. The intensities are ordered by reading across rows of the intensity matrix, starting with the first, counting the positive off-diagonal elements of the matrix.
	For example, for a multi-state model with three transition intensities, and two covariates $x$ and $y$ on each intensity,
	covariates=list(x = $c(-0.3, -0.3, -0.3)$ , y= $c(0.1, 0.1, 0.1)$ )

42

## simmulti.msm

death	Vector of indices of the death states. A death state is an absorbing state whose time of entry is known exactly, but the individual is assumed to be in an unknown transient state ("alive") at the previous instant. This is the usual situation for times of death in chronic disease monitoring data. For example, if you specify death = $c(4, 5)$ then states 4 and 5 are assumed to be death states.
	death = TRUE indicates that the final state is a death state, and death = FALSE (the default) indicates that there is no death state.
start	A vector with the same number of elements as there are distinct subjects in the data, giving the states in which each corresponding individual begins. Defaults to state 1 for each subject.
ematrix	An optional misclassification matrix for generating observed states conditionally on the simulated true states. As defined in msm.
hmodel	An optional hidden Markov model for generating observed outcomes condition- ally on the simulated true states. As defined in msm.
hcovariates	List of the same length as hmodel, defining any covariates governing the hid- den Markov outcome models. Unlike in the msm function, this should also de- fine the values of the covariate effects. Each element of the list is a named vector of the initial values for each set of covariates for that state. For example, for a three-state hidden Markov model with two, one and no covariates on the state 1, 2 and 3 outcome models respectively,
	NULL)

## Details

sim.msm is called repeatedly to produce a simulated trajectory for each individual. The state at each specified observation time is then taken to produce a new column state. The effect of time-dependent covariates on the transition intensity matrix for an individual is determined by assuming that the covariate is a step function which remains constant in between the individual's observation times. If the subject enters an absorbing state, then only the first observation in that state is kept in the data frame. Rows corresponding to future observations are deleted. The entry times into states given in death are assumed to be known exactly.

#### Value

A data frame with columns,

subject	Subject identification indicators
time	Observation times
state	Simulated (true) state at the corresponding time
obs	Observed outcome at the corresponding time, if ematrix or hmodel was supplied

plus any supplied covariates.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

sim.msm

#### Examples

sojourn.msm Mean sojourn times from a multi-state model

## Description

Estimate the mean sojourn times in the transient states of a multi-state model and their confidence limits.

## Usage

sojourn.msm(x, covariates="mean", cl=0.95)

## Arguments

Х	A fitted multi-state model, as returned by msm
covariates	The covariate values at which to estimate the mean sojourn times. This can either be:
	the string "mean", denoting the means of the covariates in the data (this is the default),
	the number 0, indicating that all the covariates should be set to zero,
	a list of values, with optional names. For example,
	list (60, 1), where the order of the list follows the order of the covariates originally given in the model formula, or a named list, e.g.
	list (age = 60, sex = 1)
cl	Width of the symmetric confidence interval to present. Defaults to 0.95.

## Details

The mean sojourn time in a transient state r is estimated by  $-1/q_{rr}$ , where  $q_{rr}$  is the rth entry on the diagonal of the estimated transition intensity matrix. Calls deltamethod to find approximate standard errors. Confidence limits are estimated by assuming normality on the log scale.

44

#### statetable.msm

## Value

A data frame with components:

estimates	Estimated mean sojourn times in the transient states.
SE	Corresponding standard errors.
L	Lower confidence limits.
U	Upper confidence limits.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

msm, qmatrix.msm, deltamethod

statetable.msm Table of transitions

## Description

Calculates a frequency table counting the number of times each pair of states were observed in successive observation times. This can be a useful way of summarising multi-state data.

# Usage

statetable.msm(state, subject, data=NULL)

## Arguments

state	Observed states, assumed to be ordered by time within each subject.
subject	Subject identification numbers corresponding to state. If not given, all observations are assumed to be on the same subject.
data	An optional data frame in which the variables represented by subject and state can be found.

## Value

A frequency table with starting states as rows and finishing states as columns.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

crudeinits.msm

## Examples

```
## Heart transplant data
data(heart)
## 148 deaths from state 1, 48 from state 2 and 55 from state 3.
statetable.msm(state, PTNUM, data=heart)
```

msm.summary

Summarise a fitted multi-state model

## Description

Summary method for fitted msm models. Currently, this produces a table of observed and expected state prevalences for each time. For models with covariates, prints hazard ratios with confidence intervals for covariate effects.

## Usage

```
## S3 method for class 'msm':
summary(object, times=NULL, timezero=NULL, initstates=NULL,
covariates="mean", misccovariates="mean", hazard.scale=1, ...)
```

## Arguments

object	A fitted multi-state model object, as returned by msm.
times	A sequence of times at which to compare observed and expected prevalences of each state. Defaults to seq(min(times), max(times), (max(times) - min(times))/10).
timezero	Initial time of the Markov process. Expected values are forecasted from here. Defaults to the minimum of the observation times given in the data.
initstates	Optional vector of the same length as the number of states. Gives the numbers of individuals occupying each state at timezero. The default is that all individuals are in state 1.
covariates	Covariate values for which to forecast expected state occupancy. See qmatrix.msm Defaults to the mean values of the covariates in the data set.
misccovariate	es
	(Misclassification models only) Values of covariates on the misclassification probability matrix for which to forecast expected state occupancy. Defaults to the mean values of the covariates in the data set.
hazard.scale	Vector with same elements as number of covariates on transition rates. Corresponds to the increase in each covariate used to calculate its hazard ratio. Defaults to all 1.
	further arguments passed to or from other methods

46

## surface.msm

## Value

A list of class summary.msm, with components:

prevalencesOutput from prevalence.msm.hazardOutput from hazard.msmhazard.scaleValue of the hazard.scale argument

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

msm,prevalence.msm, hazard.msm

surface.msm Explore the likelihood surface

## Description

Plot the log-likelihood surface with respect to two parameters.

## Usage

## Arguments

Х	Output from msm, representing a fitted msm model.
params	Integer vector with two elements, giving the indices of the parameters to vary. All other parameters will be fixed. Defaults to $c(1,2)$ , representing the first two log transition intensities. See the fixedpars argument to msm for a definition of these indices.
np	Number of grid points to use in each direction, by default 10. An np $\times$ np grid will be used to evaluate the likelihood surface. If 100 likelihood function evaluations is slow, then reduce this.
type	Character string specifying the type of plot to produce.
"contour" "filled.contour "persp" "image"	Contour plot, using the R function contour. Solid-color contour plot, using the R function filled.contour. Perspective plot, using the R function persp. Grid color plot, using the R function image.

point Vector of length n, where n is the number of parameters in the model, including

	the parameters that will be varied here. This specifies the point at which to fix the likelihood. By default, this is the maximum likelihood estimates stored in the fitted model x, x\$estimates.
xrange	Range to plot for the first varied parameter. Defaults to plus and minus two standard errors, obtained from the Hessian at the maximum likelihood estimate.
yrange	Range to plot for the second varied parameter. Defaults to plus and minus two standard errors, obtained from the Hessian at the maximum likelihood estimate.
••••	Further arguments to be passed to the plotting function.

#### **Details**

Draws a contour or perspective plot. Useful for diagnosing irregularities in the likelihood surface. If you want to use these plots before running the maximum likelihood estimation, then just run msm with all estimates fixed at their initial values.

contour.msm just calls surface.msm with type = "persp". persp.msm just calls surface.msm with type = "persp".

image.msm just calls surface.msm with type = "persp".

As these three functions are methods of the generic functions contour, persp and image, they can be invoked as contour (x), persp (x) or image (x), where x is a fitted msm object.

#### Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

#### See Also

msm, contour, filled.contour, persp, image.

tnorm

Truncated Normal distribution

#### Description

Density, distribution function, quantile function and random generation for the truncated Normal distribution with mean equal to mean and standard deviation equal to sd before truncation, and truncated on the interval [lower, upper].

#### Usage

```
dtnorm(x, mean=0, sd=1, lower=-Inf, upper=Inf, log = FALSE)
ptnorm(q, mean=0, sd=1, lower=-Inf, upper=Inf, lower.tail = TRUE, log.p = FALS
qtnorm(p, mean=0, sd=1, lower=-Inf, upper=Inf, lower.tail = TRUE, log.p = FALS
rtnorm(n, mean=0, sd=1, lower=-Inf, upper=Inf)
```

#### tnorm

#### Arguments

x,q	vector of quantiles.
р	vector of probabilities.
n	number of observations. If $length(n) > 1$ , the length is taken to be the number required.
mean	vector of means.
sd	vector of standard deviations.
lower	lower truncation point.
upper	upper truncation point.
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are $P[X \le x]$ , otherwise, $P[X > x]$ .

## Details

The truncated normal distribution has density

$$f(x,\mu,\sigma) = \phi(x,\mu,\sigma) / (\Phi(u,\mu,\sigma) - \Phi(l,\mu,\sigma))$$

for  $l \le x \le u$ , and 0 otherwise.

 $\mu$  is the mean of the original Normal distribution before truncation,

 $\sigma$  is the corresponding standard deviation,

u is the upper truncation point,

*l* is the lower truncation point,

 $\phi(x)$  is the density of the corresponding normal distribution, and

 $\Phi(x)$  is the distribution function of the corresponding normal distribution.

If mean or sd are not specified they assume the default values of 0 and 1, respectively.

If lower or upper are not specified they assume the default values of -Inf and Inf, respectively, corresponding to no lower or no upper truncation.

Therefore, for example, dtnorm(x), with no other arguments, is simply equivalent to dnorm(x).

Only rtnorm is used in the msm package, to simulate from hidden Markov models with truncated normal distributions. These functions are merely provided for completion, and are not optimized for numerical stability. To fit a hidden Markov model with a truncated Normal response distribution, use a hmmTNorm constructor. See the hmm-dists help page for further details.

## Value

dtnorm gives the density, ptnorm gives the distribution function, qtnorm gives the quantile function, and rtnorm generates random deviates.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

dnorm

## Examples

```
x <- seq(50, 90, by=1)
plot(x, dnorm(x, 70, 10), type="l", ylim=c(0,0.06)) ## standard Normal distribution
lines(x, dtnorm(x, 70, 10, 60, 80), type="l") ## truncated Normal distribution</pre>
```

totlos.msm Total length of stay

## Description

Estimate the expected total length of stay in each transient state, for a given period of evolution of a multi-state model. This assumes that the transition rates do not change with time.

## Usage

totlos.msm(x, start=1, fromt=0, tot=Inf, covariates="mean", ci.boot=FALSE, cl=0.95,

## Arguments

х	A fitted multi-state model, as returned by msm.
start	State at the beginning of the period.
fromt	Time from which to estimate total length of stay. Defaults to 0, the beginning of the process.
tot	Time up to which total length of stay is estimated. Defaults to infinity, giving the expected time spent in the state until absorption. For models without an absorbing state, $t$ must be specified.
covariates	The covariate values to estimate for. This can either be:
	the string "mean", denoting the means of the covariates in the data (this is the default),
	the number 0, indicating that all the covariates should be set to zero,
	or a list of values, with optional names. For example list (60, 1)
	where the order of the list follows the order of the covariates originally given in the model formula, or a named list,
	list (age = 60, sex = 1)
ci.boot	Calculate a bootstrap confidence interval. This is usually time-consuming, and disabled by default. See boot.msm for more details of bootstrapping in <b>msm</b> .
cl	Width of the symmetric confidence interval
В	Number of bootstrap replicates
	Further arguments to be passed to the integrate function to control the numerical integration.

50

#### transient.msm

#### Details

The expected total length of stay in state j between times  $t_1$  and  $t_2$ , from the point of view of an individual in state i at time 0, is defined by the integral from  $t_1$  to  $t_2$  of the i, j entry of the transition probability matrix P(t). As the individual entries of  $P(t) = \exp(tQ)$  are not available explicitly in terms of t for a general Markov model, this integral is calculated numerically, using the integrate function. This may take a long time for models with many states where P(t) is expensive to calculate.

For a model where the individual has only one place to go from each state, and each state is visited only once, for example a progressive disease model with no recovery or death, these are equal to the mean sojourn time in each state. However, consider a three-state health-disease-death model with transitions from health to disease, health to death, and disease to death, where everybody starts healthy. In this case the mean sojourn time in the disease state will be greater than the expected length of stay in the disease state. This is because the mean sojourn time in a state is conditional on entering the state, whereas the expected total time diseased is a forecast for a healthy individual, who may die before getting the disease.

## Value

A vector of expected total lengths of stay for each transient state.

#### Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## See Also

sojourn.msm, pmatrix.msm, integrate, boot.msm.

transient.msm Transient and absorbing states

#### Description

Returns the transient and absorbing states of either a fitted model or a transition intensity matrix.

#### Usage

```
transient.msm(x=NULL, qmatrix=NULL)
absorbing.msm(x=NULL, qmatrix=NULL)
```

#### Arguments

Х	A fitted multi-state model as returned by msm.
qmatrix	A transition intensity matrix. The diagonal is ignored and taken to be minus the
	sum of the rest of the row.

## Value

A vector of the ordinal indices of the transient or absorbing states.

## Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

viterbi.msm Calculate the most likely path through underlying stages

## Description

For a fitted hidden Markov model, the Viterbi algorithm recursively constructs the path with the highest probability through the underlying stages.

## Usage

viterbi.msm(x)

#### Arguments

Х

A fitted hidden Markov multi-state model, as produced by msm

#### Value

A data frame with columns:

subject = subject identification numbers

time = times of observations

observed = corresponding observed states

fitted = corresponding fitted states found by Viterbi recursion. If the model is not a hidden Markov model, this is just the observed states.

#### Author(s)

C. H. Jackson (chris.jackson@imperial.ac.uk)

## References

Durbin, R., Eddy, S., Krogh, A. and Mitchison, G. *Biological sequence analysis*, Cambridge University Press, 1998.

#### See Also

msm

# Index

```
*Topic datasets
    aneur, 3
    bos.5
    fev, 11
    heart, 13
    psor, 36
*Topic math
    deltamethod, 8
    MatrixExp, 1
*Topic models
    coef.msm, 6
    ematrix.msm, 10
    hazard.msm, 12
    logLik.msm, 16
    msm, 19
    odds.msm, 29
    plot.msm, 31
    prevalence.msm, 35
    qmatrix.msm, 37
    gratio.msm, 39
    sim.msm, 40
    simmulti.msm, 41
    sojourn.msm, 43
    surface.msm, 47
    viterbi.msm, 52
absorbing.msm(transient.msm), 51
aneur, 3
boot.msm, 4, 4, 32, 33, 50, 51
bos.5
coef.msm, 6
contour, 47, 48
contour.msm (surface.msm), 47
crudeinits.msm, 7, 45
```

dbinom, 15 deltamethod, 8, 39, 44 dexp, 15, 30

dgamma, 15 dmenorm (medists), 17 dmeunif (medists), 17 dnbinom, 15 dnorm, 19, 49 dpexp (pexp), 29 dpois, 15 dtnorm, 19 dtnorm (tnorm), 48 dunif,19 dweibull, 15 ematrix.msm, 10, 11, 26, 39 fev. 11 filled.contour, 47, 48 hazard.msm, 12, 29, 46, 47 heart, 13 hmm-dists, 18, 21, 49 hmm-dists, 14 hmmBinom (hmm-dists), 14 hmmCat, 21 hmmCat (hmm-dists), 14 hmmExp(hmm-dists), 14 hmmGamma (hmm-dists), 14 hmmIdent (hmm-dists), 14 hmmLNorm (hmm-dists), 14 hmmMETNorm, 18 hmmMETNorm (hmm-dists), 14 hmmMEUnif, 18 hmmMEUnif (hmm-dists), 14 hmmNBinom (hmm-dists), 14 hmmNorm (hmm-dists), 14 hmmPois (hmm-dists), 14 hmmTNorm, 49 hmmTNorm (hmm-dists), 14 hmmUnif (hmm-dists), 14 hmmWeibull (hmm-dists), 14

image, 47, 48

### INDEX

image.msm(surface.msm), 47 integrate, 50, 51 load.4 logLik.msm, 16 MatrixExp, 1 medists, 14, 17 msm, 4, 6-8, 10-13, 16, 17, 19, 27, 29, 31-33, 35, 36, 38, 39, 42, 44, 46–48, 50–52 msm.summary, 46 nlm, 25, 27 odds.msm, 13, 29 optim, 24, 25, 27 persp, 47, 48 persp.msm (surface.msm), 47 pexp, 29 plot.msm, 28, 31 pmatrix.msm, 4, 5, 27, 28, 32, 33, 34, 39, 51 pmatrix.piecewise.msm, 33, 33 pmenorm (medists), 17 pmeunif (medists), 17 ppexp(pexp), 29 prevalence.msm, 35, 46, 47 print.summary.msm(msm.summary), 46 psor, 36 ptnorm (tnorm), 48 qmatrix.msm, 11, 26-28, 33, 35, 37, 38, 40, 44,46 qmenorm (medists), 17 qmeunif (medists), 17 qpexp (pexp), 29 qratio.msm, 39 qtnorm (tnorm), 48 rmenorm (medists), 17 rmeunif(medists), 17 rpexp(pexp), 29 rtnorm (tnorm), 48 save,4 sim.msm, 30, 40, 42, 43 simmulti.msm, 28, 41, 41 sojourn.msm, 27, 28, 39, 43, 51

statetable.msm, 8, 45
summary.msm, 12, 13, 28, 36
summary.msm (msm.summary), 46
surface.msm, 47

tnorm, 14, 48
totlos.msm, 5, 50
transient.msm, 51

viterbi.msm, 36, 52

## 54