1 Modelo Poisson

$$y_{ij} \sim Poisson(\lambda_i)$$

$p(\theta|y) \propto p(y|\theta)p(\theta)$,

onde $\theta = (u_i, \alpha, \beta, \tau^2)$

$p(\theta|y) \propto \prod_{i=1}^{10} \prod_{j=1}^{20} exp\left\{y_{ij}log\lambda_i - \lambda_i - logy_{ij}!\right\} \prod\limits_{i=1}^{10} p(u_i, \alpha, \beta, \tau^2)$

$p(\theta|y) \propto \prod_{i=1}^{10} \prod_{j=1}^{20} exp\left\{y_{ij}log\lambda_i - \lambda_i - logy_{ij}!\right\} \prod_{i=1}^{10} p(u_i, |\alpha, \beta, \tau)p(\alpha)p(\beta)p(\tau^2)$

Onde, $u_i \sim N(\alpha + \beta_i, \sigma^2)$,

$p(\alpha) \sim N(0, \tau_\alpha)$,

$p(\beta_i) \sim N(0, \tau_\beta)$,

$p(\tau^2) \sim G(A, B)$,$\tau^2 = \sigma^{-2}$

2 Modelo Geoestatistico

$z_i \sim N(\mu_i, \sigma^2)$

$p(\theta|y) \propto p(y|\theta)p(\theta)$,

onde $\theta = (\mu, \beta, \beta_1, \alpha)$

$p(\theta|y) \propto \prod_{i=1}^{50} \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{\frac{1}{2\sigma^2}(y_i - \mu_i)^2\right\} p(\mu, \beta, \beta_1, \alpha)$

$p(\theta|y) \propto \prod_{i=1}^{50} \frac{1}{\sqrt{2\pi\sigma^2}} exp\left\{\frac{1}{2\sigma^2}(y_i - \mu_i)^2\right\} p(\mu, |\beta, \beta_1, \alpha)p(\beta)p(\beta_1)p(\alpha)$

Onde, $\mu \sim NM(\alpha + \beta x + \beta_1 y, \tau^{-2}R(\phi, \kappa))$,

$p(\phi) \sim U(a, b)$,

$p(\beta) \sim N(0, \tau_\beta)$,

$p(\beta_1) \sim N(0, \tau_{\beta_1})$,

$p(\alpha) \sim N(0, \tau_\alpha)$,

$p(\tau) \sim G(A, B)$,

$\tau = \sigma^{-2}$

$\mu = (\mu_1, \ldots, \mu_{50})$, $x = (x_1, \ldots, x_{50})$, e $y = (x_1, \ldots, y_{50})$

3 Modelo Univariado Espaço Temporal

$Y_{it} \sim Poisson(\mu_{it})$,$\mu_{it} = e_{it}\psi_{it}$

$e_{it} = p_{it}p_t*$, $p_t* = \dfrac{\sum_i y_{it}}{\sum_i p_{it}}$,onde p_{it} é a população do município i no ano t

$log(\psi_{it}) = \alpha_t + \beta_t x_{it} + \phi_{it}$

$$\phi_i|\phi_j, j \neq i \sim N\left(\frac{\sum_{j\in\delta_i} w_{ij}\phi_j}{\sum_{j\in\delta_i} w_{ij}}, \frac{1}{\tau^2 \sum_{j\in\delta_i} w_{ij}}\right)$$

$$p(\phi|\tau^2) \propto \frac{1}{\tau^n} exp\left\{-\frac{1}{2\tau^2}\sum_{i=1}^{n}\sum_{i<j} w_{ij}(\phi_i - \phi_j)^2\right\}$$

$\beta_t = \beta_{t-1} + w_t$, $w_t \sim N(0, \sigma_b^2)$

$p(\theta|y) \propto p(y|\theta)p(\theta)$,

onde ,$\theta = (\alpha, \beta, \phi, \tau^2, \sigma_b^2)$

$\alpha = (\alpha_1, \ldots, \alpha_T)$, $\beta = (\beta_1, \ldots, \beta_T)$, $\phi = (\phi_1, \ldots, \phi_t)$,$\phi_t = (\phi_{1t}, \ldots, \phi_{nt})$

$$p(\theta|y) \propto \prod_{i=1}^{N}\prod_{t=1}^{T} exp\left\{y_{it}log(e_{it}\psi_{it}) - e_{it}\psi_{it} - logy_{it}!\right\}\prod_{t=1}^{T} p(\alpha_t, \beta_t, \phi_t, \tau^2, \sigma_b^2)$$

$$p(\theta|y) \propto \prod_{i=1}^{N}\prod_{t=1}^{T} exp\left\{y_{it}log(e_{it}\psi_{it}) - e_{it}\psi_{it} - logy_{it}!\right\}\prod_{t=1}^{T} p(\alpha_t), p(\beta_t|\beta_{t-1}, \sigma_b^2)p(\phi_t, |\tau^2)$$

$p(\sigma_b^2)p(\beta_0)p(\tau^2)$

$p(\tau^2) \sim IG(A, B)$,

$p(\sigma_b^2) \sim IG(A, B)$,

$p(\beta_0) \sim N(0, \tau_{\beta_0})$

4 Modelo Multivariado Espacial

$Y_{ik} \sim Poisson(\mu_{ik})$, $\mu_{ik} = e_{ik}\psi_{ik}$

$log(\mu_{ik}) = log(e_{ik}) + \alpha_k + \phi_{ik}$

$p(\Phi) \propto exp\left\{-\frac{1}{2}\Phi'[\Lambda\Theta(D-W)]\Phi\right\}$

$\frac{\phi_{i1}}{\phi_{i2}}|\phi_{-(i1,i2)} \sim N(\frac{\phi_{i1^*}}{\phi_{i2^*}}, (w_{i+}\Lambda)^{-1})$

$\phi_{i1^*} = \sum_j w_{ij}\phi_{j1}/w_{i+}$, $\phi_{i2^*} = \sum_j w_{ij}\phi_{j2}/w_{i+}$

Λ é uma matriz 2x2 positiva definida ,

W é uma matriz simétrica onde é especificado os pesos da vizinhança $w_{ij} = 1$ se a área geográfica faz fronteira $w_{ij} = 0$, caso contrário

$D = diag(w_{i_+})$, $w_{i+} = \sum_j w_{ij}$

Θ é o produto de Kronecker

onde ,$\theta = (\alpha, \sigma_1^2, \sigma_2^2, \sigma_b^2, \rho, \Phi)$,$\Phi = (\phi_1, \phi_2)$, $\phi_1 = (\phi_{11}, \ldots, \phi_{n1})$, $\phi_2 = (\phi_{21}, \ldots, \phi_{n2})$

$p(\theta|y) \propto p(y|\theta)p(\theta)$,

$p(\theta|y) \propto \prod_{i=1}^{N}\prod_{k=1}^{K} exp\left\{y_{ik}log(e_{ik}\psi_{ik}) - e_{ik}\psi_{ik} - logy_{ik}!\right\} \prod_{k=1}^{K} p(\alpha_k, \sigma_1^2, \sigma_2^2, \sigma_b^2, \rho, \Phi)$